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Bifurcation in a Bloch-Maxwell Model for the He-Ne Laser

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This paper studies chaotic behavior in He-Neon laser models. Transition to chaos is based on considering the resonance between the laser cavity frequency and atomic cavity TEM modes. It is reflected by the Maxwell-Bloch equations as given by Arrechi [1] and Haken [2]. The coupling of the fundamental cavity mode, E with the collective variables P and Δ, that represent the atomic polarization and the population inversion, gives the following equations.

$$\dot{E} = -kE + gP$$

$$\dot{P} = -\gamma_{\perp} P + gE\Delta$$

$$\dot{\Delta} = -\gamma_{\|}(\Delta - \Delta_o) - 4gPE$$

For the parameter values $k = \sigma$, $\gamma_{\perp} = g^2/k = 1$, $g^2\Delta_o/k = r$, $\gamma_{\|} = b$, the system can be transformed into the Lorenz system about the equilibrium point $\Delta = \Delta_o$ by setting $x = E$, $y = gP/k$,$z = \Delta_o - \Delta$. The meaning of the parameters in the original equations are given by Arrechi, while σ, r, b are the Lorenz parameters.

The Maxwell-Bloch equations have more parameters than the Lorenz system; this justifies a more detailed parameter study. Chaotic behavior has been experimentally observed in laser systems[1, 2] and controlling chaos is important in obtaining laser based standards in metrology. A parameter study that would reveal the range of parameters for which chaotic behavior characterized by the well known invariant, a positive maximal Liapunov exponent would thus be of interest. Results of such a study using the Wolf algorithm.[3] will be reported. Where possible, the bifurcation mechanism that characterizes the transition to chaos is also studied by the MATCONT[4] package and Hopf bifurcation is identified in several instances.

An example is $k = 11.75$, $\gamma_{\perp} = 2.66$, $\gamma_{\|} = 2.75$, $\Delta_o = 28$, $g = 6.06$. These parameters correspond to far infrared lasers where Lorenz like chaos has been observed.

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[3] A. Wolf, J. B. Swift, H. L. Swinney and J. A. Vastano "Determining Lyapunov exponents from a time series", *Physica D* **16**, 285-317 (1985).

[4] A. Dhooge, W. Govaerts, Yu.A. Kuznetsov, W. Mestrom, A.M. Riet and B. Sautois, "MATCONT and CL_MATCONT: Continuation toolboxes in matlab", Ghent and Utrecht Universities Preprint, (2006).