Frenkel Kontorova Models, Free Energy Recursions and Burgers Shocks

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The Frenkel Kontorova Model is a classical infinite chain of atoms linked by elastic springs with equilibrium spacing a that is subject to an external periodic potential of period b [1]. The model is characterized by the competition of two different length scales, a and b. The lowest energy configurations of the chain have a very complex depedence on a/b and the relative strength of the external potential, giving rise to commensurate or incommensurate configurations, and transitions between these, as the parameters of the model are varied [2]. The ground state configurations are also closely related to the unstable (hyperbolic) orbits of 2 dimensional hamiltonian maps such as the standard map.

It is possible to approach this problem from a statistical mechanical point of view, constructing a transfer matrix that captures the evolution of the free-energy. The recursion equations for the evolution of the free-energy were derived by Griffiths and Chou for the zero-temperature case [3] and for the general case by Feigelman [4]. One of the major advantages of the transfer matrix description is that besides the inclusion of non-zero temperature, this approach readily accomodates the case of random as opposed to periodic external potentials.

It has been recently realized that a continuum hydrodynamic type evolution underlies the discrete free energy recursions. For the case of an elastic chain of particles embedded in an external potential, this evolution turns out to be governed by an iterated Burgers Equation [5, 6] and the emerging shock discontinuities have a natural interpretation in terms of meta-stable states.

In this talk, I will present the connections of FK type models and the Burgers-type evolution and illustrate this by means of an exactly solvable (non-trivial) model that we worked out recently [7].

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