
Detecting causality in multivariate time series

D. Kugiumtzis*

Department of Mathematical, Physical and Computational Sciences,
Faculty of Engineering, Aristotle University of Thessaloniki, Greece

* Electronic Address: dkugiu@gen.auth.gr

In the analysis of multiple time series the main interest is to identify the inter-dependencies among the different variables or subsystems corresponding to the time series. Besides the form and strength of inter-dependence, given as cross-correlation, coherence, coupling or synchronization, it is often important to identify its direction, a term often referred to as Granger causality. One should be aware that pair dependencies may be caused by the effect of other variables or systems. Distinguishing direct from indirect causal effects when more than two time series are available is the topic of this paper.

Linear analysis has provided appropriate tools to deal with Granger causality, e.g. improvement of fit with vector autoregressive over autoregressive model for the time domain and directed coherence for the frequency domain. For direct causal effects, the most popular measure is the Partial Directed Coherence (PDC) [1, 2]. Measures for Granger causality have been developed recently based on nonlinear dynamics and information theory, such as the transfer entropy (TE) [3]. We have also proposed very recently a causality measure derived directly from a mixed embedding scheme based on conditional mutual information criterion, which we call mutual information from mixed embedding (MIME) [4]. In this paper, we extend the measures of TE and MIME to be able to identify direct causal effects, and we name the Partial TE (PTE) and Partial MIME (PMIME).

We compare the proposed measures PTE and PMIME to each other and also to the linear counterpart PDC on some known systems, such as vector autoregressive systems and the Henon coupled maps. Further, we apply the three measures on two real-world applications, multi-channel scalp EEG recordings before, during and after epileptic seizure, and world market indices from some representative countries. A main drawback of the nonlinear measures PTE and PMIME is their inability to account for the effect of many other systems (beyond the assumed driving and response system). Also, PTE turns out to be more biased towards detecting direct causal effects (often giving spurious results), while for PMIME this problem can be treated by adjusting appropriately a free parameter. We discuss the performance of the three measures also in view of the results on the real data.

Thank you very much for following our guidelines!

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