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Analysis of Metastable States in the Infinite-Range-Interaction Inertial Heisenberg Ferromagnet

Fernando D. Nobre*

Centro Brasileiro de Pesquisas Físicas
Rua Xavier Sigaud 150
22290-180 Rio de Janeiro - RJ Brazil
* Electronic Address: fdnobre@cbpf.br

Hamiltonian systems composed by a large number of microscopic elements – interacting through long-range couplings – may pose difficulties in the applicability of the Boltzmann-Gibbs (BG) statistical-mechanics formalism. These types of systems may present a non-extensive behavior in some of its thermodynamic properties, a zero maximum Lyapunov exponent, and breakdown of ergodicity, which represent essential conditions within the standard BG formalism. Herein we discuss a simple model that exhibits such characteristics, i.e., the infinite-range-interaction inertial classical Heisenberg ferromagnet. This model consists in a modification of the well-known Heisenberg model, where the spins are replaced by classical rotators, defined by the Hamiltonian,

$$H = K + V = \frac{1}{2}\sum_{i=1}^{N}\sum_{\mu=1}^{3} L_{i\mu}^2 + \frac{1}{2N}\sum_{i,j=1}^{N}(1 - \vec{S}_i.\vec{S}_j) \ ,$$

where the index μ ($\mu = 1, 2, 3$) denotes Cartesian components and $L_{i\mu}$ represents the μ-component of the angular momentum (or the rotational velocity, since we are assuming unit inertial moments) of rotator i. The usual canonical-ensemble mean-field solution of the inertial classical Heisenberg ferromagnet is briefly reviewed, showing the well-known second-order phase transition. This model is studied numerically within the microcanonical ensemble, through molecular dynamics. The time evolution of the kinetic temperature indicates that certain basins of attraction exist for the initial conditions, for which the system evolves into a metastable state, before attaining the terminal thermal equilibrium. Some properties of these metastable states are studied and are shown to exhibit curious behavior: (i) Different initial conditions may lead to distinct metastable states; (ii) They are characterized by durations that diverge as $N \to \infty$, implying that the system should remain forever in such states in the thermodynamic limit; (iii) The maximum Lyapunov exponent decreases with N like $\lambda_{\max} \sim N^{-\kappa}$ ($\kappa > 0$); (iv) The angular-velocity distributions computed from time intervals over a single sample, or from averages over samples at a fixed time (i.e., ensemble averages), may present very different behavior; (v) For some initial conditions, the time-averaged angular-velocity distributions are characterized by long tails.