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Time–Evolving Statistics of Chaotic Orbits of Conservative Maps in the Context of Central Limit Theorem

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The existence of a central limit theory for deterministic variables has been known for some time [1]. More recently, this theory has been the focus of attention from a statistical mechanics viewpoint, aiming to determine the probability density function (pdf) of systems characterized by non-additive entropy S_q [2, 3]. In the past, this direction has been pursued using the distribution of the sum of deterministic variables for some special kind of correlation [4], or even for systems evolving in metastable states [5].

In the present work, we study numerically the pdf of sums of $N \to \infty$ iterates of the conservative (area-preserving) 2–dimensional perturbed Mc Millan map [6], which may be interpreted as describing the effect of a simple linear focusing system supplemented by a periodic sequence of thin nonlinear lenses:

$$\begin{cases} x_{n+1} = y_n \\ y_{n+1} = -x_n + 2\mu \frac{y_n}{1+y_n^2} + \epsilon y_n \end{cases} \tag{1}$$

where ϵ, μ are real parameters. In the spirit of central limit theory, we have analyzed the pdf of shifted and normalized sums of the x-component of (1) in "thin" chaotic layers of the x, y phase plane and discovered that they exhibit a very interesting time evolution, for a wide range of parameters (ϵ, μ): In particular, they appear to exhibit "generically" *three stages*, passing from a *q-Gaussian*, to a *triangular* shape and finally to the well–known *Gaussian* form. These stages accompany a slow diffusion process through which the orbits move successively to wider chaotic domains, where the dynamics is more uniformly ergodic and Boltzmann–Gibbs statistics is expected to prevail. Preliminary results by us on a 4–dimensional volume–preserving map, as well as by other researchers on multi–dimensional Hamiltonian systems [7] show similar time–evolving statistics for chaotic orbits diffusing slowly into regimes of more uniformly random dynamics.

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