Abstract for GR-TR Conference on Statistical Mechanics and Dynamical Systems

Talk Invited | Invited Talk

Nonlinear stochastic differential equations and $1/f$ noise

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Power-law distributions of spectra of signals, including $1/f$ noise, as well as scaling behavior in general, are ubiquitous in physics and in many other fields, including natural phenomena, human activities, traffics in computer networks, and financial markets. This subject has been a hot research topic for many decades. Despite the numerous models and theories proposed since its discovery more than 80 years ago, the subject of $1/f$ noise remains still open for new discoveries.

Here we analyze non-linear stochastic differential equations (SDEs) [1, 2] generating signals with $1/f$. The general form of the SDE is

$$\mathrm{d}x = \sigma^2(\eta - \lambda/2)x^{2\eta-1}\mathrm{d}t + \sigma x^{\eta}\mathrm{d}W_t \tag{1}$$

As a particular case of the proposed SDEs and its modifications are Constant Elasticity of Variance Process, Bessel Process and Squared Bessel Process, which are used for modeling of the financial markets. The power spectral density of the signal generated by SDE (1) is related to the behavior of the eigenvalues of the corresponding Fokker-Planck equation [3]. The proposed SDEs can be modified to get both q-Gaussian distributions of signal intensity, featured in nonextensive statistical mechanics, and $1/f$ behavior of the power spectral density [4].

We also present a couple of models leading to the nonlinear SDE (1). One of them is a point process where the inter-event time stochastically diffuses in a wide region. Another is a simple agent model of herding, providing a microscopic mechanism of the macroscopic description by SDE (1) [5].

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