

## Renormalization of Random Multiplicative Processes

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We consider a random multiplicative process as a minimal model of complex system's growth phenomena. It is known that a single random multiplicative process produces a power law steady distribution in general, so that, this simple model can be used as a first step model for phenomena in which variables follow power law distributions. Starting with this simple model of independent variables, we consider sum of many variables. It can be shown that the sum satisfies the same form of random multiplicative stochastic process with modified random variables for multiplication. Namely, by this grouping we can define a renormalization of random multiplicative variables. By numerical simulation and theoretical analysis we can show that starting by repeating renormalization processes we have a universal limit distribution of random multiplier. The distribution is given by Lorentz distribution having power law tails. This result implies that even in the case that microscopic growth rate distribution has no long tail, renormalized macroscopic growth rate distribution has power law tails in general, so that a macroscopic system generally has a risk of extraordinarily large fluctuations. We apply this model to company's financial data such as sales or income with the multiplier representing annual growth rate. This model gives a consistent view to the power law distribution of sales and long-tailed distribution of growth rate observed in real company data.