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Scale invariant probabilistic model having two dimensional $q-$Gaussians as $N \to \infty$ limiting distribution

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It has been recently introduced in [1] and further generalized in [2] a family of one dimensional scale-invariant probabilistic models characterized by a real number $\nu > 0$ having one dimensional $q-$Gaussians as $N \to \infty$ limiting distributions based on the so called Leibniz triangle [3], and on the Pascal triangle. The model consisted of a set of N *equal, long-range-correlated binary* random variables —corresponding to the binomial distribution in the uncorrelated case— in which scale-invariant correlations were introduced.

In this contribution we generalize the model by allowing the N discrete random variables to take on three different values —hence the trinomial distribution is obtained in the uncorrelated limit—. In addition, we introduce the so called Pascal pyramid, as well as what we may call the *Leibniz pyramid*, given by

$$r^{(1)}_{N,n,m} = \frac{2}{(N+2)(N+1)\binom{N}{n\,,m}} \tag{1}$$

where n, m and N are positive integers with $0 \leq n + m \leq N$, and $\binom{N}{n\,,m}$ are the trinomial coefficients.

By properly rescaling certain subpyramids of the Leibniz pyramid (1) we get a family of pyramids characterized by a real number $\nu \geq 1$, given by

$$r^{(\nu)}_{N,n,m} = \frac{B(n+\nu, m+\nu)B(n+m+2\nu, N-n-m+\nu)}{B(\nu,\nu)B(\nu,2\nu)} \tag{2}$$

which follow the *generalized Leibniz rule*

$$r^{(\nu)}_{N,n,m} + r^{(\nu)}_{N,n+1,m-1} + r^{(\nu)}_{N,n,m-1} = r^{(\nu)}_{N-1,n,m-1} \tag{3}$$

responsible for the scale-invariant character of the correlations; by $B(x, y)$ we note the Beta function. The actual probabilities of the model are given by

$$P^{(\nu)}_{N,n,m} = \binom{N}{n\,,m} r^{(\nu)}_{N,n,m} \tag{4}$$

The main result of our contribution states that under an appropiate change of variables, the limiting $N \to \infty$ probability distribution of the model is a two-dimensional $q-$Gaussian with a value of q given by

$$q_\nu = \frac{\nu-2}{\nu-1} \tag{5}$$

[1] A. Rodríguez, V. Schwammle and C. Tsallis, JSTAT P09006 (2008).

[2] R. Hanel, S. Thurner and C. Tsallis, Eur. Phys. J. B **72**, 263 (2009).

[3] G. Polya, *Mathematical Discovery*, Vol. 1, page 88 (John Wiley and Sons, New York, 1962).